

CAPITAL ASSET PRICING MODEL (CAPM)

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Abstract

The Capital Asset Pricing Model (CAPM) is a market equilibrium model used to define the existing trade off between risk and expected return in portfolio choices. The CAPM attempts to answer the questions that come from the Markowitz's mean-variance approach, and the paper describes the model and its measurement.

The CAPM is based on a theoretical scheme to concretely assess the risk connected to a certain level of return according to the individual utility function. The CAPM attempts to answer the questions that come from the Markowitz's mean-variance approach, where investors make an optimal portfolio in accordance with the rule of a greater return for an equal risk (variance), or a lower risk with an equal return.

The model hypotheses, already seen in the Markowitz model, concern the behaviour of the individual and that of the market:

- The investors want to maximise their final wealth and they are risk averse.
- The investment period is unique and forecasts are formulated at the beginning of the period.
- The expected return and the standard deviation are the only parameters that orientate the portfolio choice.

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- The assets are perfectly divisible and there are no transaction costs or taxes.
- The market is atomistic, there are no barriers with regard to investment possibilities, and all investors have the same opportunities even if the available amount of wealth differs between them.

Moreover, it is necessary to add further assumptions to these hypotheses that are essential in establishing the concave frontier:

- All securities are negotiable.
- The market is perfect; that is, the information is freely and immediately available for all investors.
- Investor expectations are homogeneous, therefore each of them has the same perception concerning the expected return, the variances and the co-variances: the efficient frontier is unique and the same for all investors.
- The possibility to grant or obtain unlimited loans at a unique free-risk interest rate.

The previous hypotheses allow investors, by knowing expected returns, standard deviation of securities and covariance among securities, to draw the same concave set (in virtue of the homogenous expectations) and to choose the efficient portfolios. Since the CAPM is an equilibrium market model, each investor will make the same choice.

This framework implies that the expected return connected to a specific security shows a linear relationship with the existing co-variance between this return and the market portfolio return.

In the Sharpe and Lintner version, the principal equation is:

$$E[R_i] = R_f + \beta_{im} (E[R_m] - R_f),$$

$$\text{where } \beta_{im} = (\text{Cov}[R_i, R_m]) / (\text{Var}[R_m])$$

In the previous formulation R_i is the asset return; R_m is the market return and R_f is the free-risk rate.

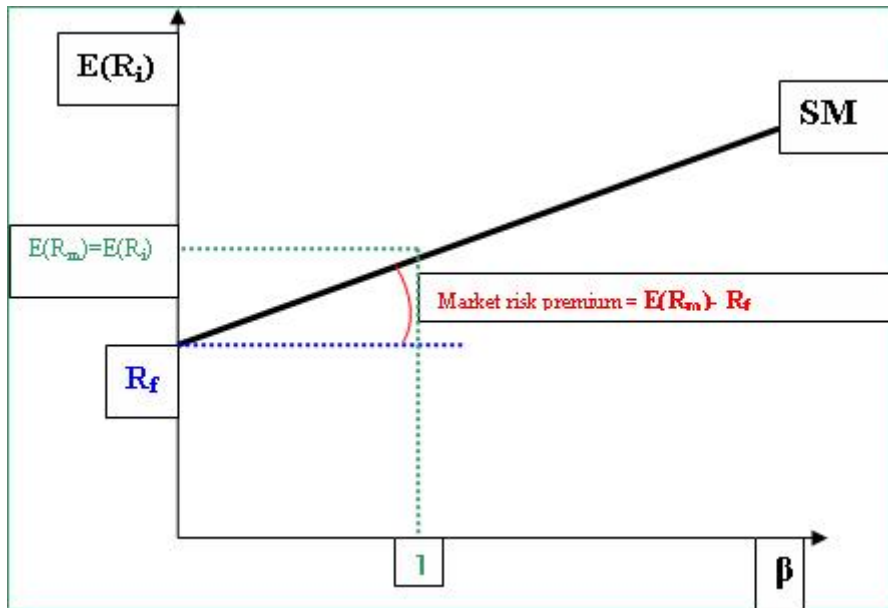


Figure: graphic representation of the CAPM

The same concept could be simply expressed in terms of excess return over free-risk rate. Calling Z_i the excess return (effectively realised) of the asset i over the free-risk rate, then inserting $Z_i = [R_i - R_f]$, and introducing the expected value of the variables Z_i and Z_m , the fundamental equations of the CAPM will become:

$$E[Z_i] = \beta_{im} E[Z_m], \text{ with } \beta_{im} = \frac{\text{Cov}[Z_i, Z_m]}{\text{Var}[Z_m]} .$$

In the previous equation, Z_m represents the excess return over the market portfolio of the various assets. Assuming that the risk-free rate is not stochastic (then its expected value is exactly equal to R_f), the two different equation versions coincide.

By using the CAPM, it is possible to define an asset value with respect to its non-diversifiable risk (β). The CAPM is based on the following insight: in a competitive market, the expected risk premium may vary proportionally in accordance with Beta. The expected risk premium of an investment with Beta equal to 0.5 is equal to half of the market's expected risk premium. Furthermore, the expected risk premium of an investment for which Beta is equal to 2.0 is double the market's expected risk premium. More specifically, CAPM is a one-factor model where the factor is coefficient β . In general, the portfolio risk premium is a linear function of β and of the market portfolio risk premium. Therefore, there will be a single asset risk premium equal to $\beta \cdot (R_m - R_f)$.

It is then possible to observe:

- A zero value β corresponds to a free-risk asset where the return is R_f .
- A β value of one corresponds to the market portfolio return. In this situation, the expected return will be R_m .

The line that links the asset expected return and its β has a positive inclination: it is easy to note a positive correlation between the sensitivity to systematic risk, measured by β , and the expected return (this is the so-called **Security Market Line (SML)** to which, in equilibrium, all the bought/sold stocks in the market belong). In other words, a progressive risk increase corresponds to a higher expected return. The CAPM is not the only reference model used for risk/return analysis. Other models are the *Arbitrage Pricing Model (APM)*, where the fundamental hypothesis is that all investors with the same risk exposure should be analysed by using the same price (no arbitrage), and where the measure of market risk Beta is given by various (non specified) market risk factors. Then, there is the *Multi-Factorial model*, which has the same hypothesis of the APM, and where Beta is measured with respect to various macroeconomic factors. There are different versions of the CAPM, such as the *intertemporal-CAPM*, the *Consumption-CAPM*, and the *Multi-CAPM* (the most

famous example is the one with the Fama-French factors), which are all evolutions of the basic model and where it is possible to add explicative variables in order to better capture market risk.

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