

VALUE AT RISK (VaR)

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Abstract

*The value at risk (VaR) measures the **risk** of loss associated to **financial assets**. For a given time period (normally ranging from 1 to 10 days), and with a given probability confidence (generally equal to 95% or 99%); this measure represents the maximum loss the investor can suffer when holding financial assets. The time horizon used to calculate the VaR depends on the investment duration; the value at risk is used to compute the minimum capital requirements necessary to compensate losses resulting from market risk, according to the BIS banking regulation.*

Value at risk (VaR) measures the risk of loss associated to financial assets. For a given time period (normally ranging from 1 to 10 days), and with a given probability level (generally equal to 95% or 99%), this measure represents the maximum loss the investor can suffer when holding financial assets. This loss derives from a model implementation, and reflects the interaction of a number of factors assumed correlated one with the other.

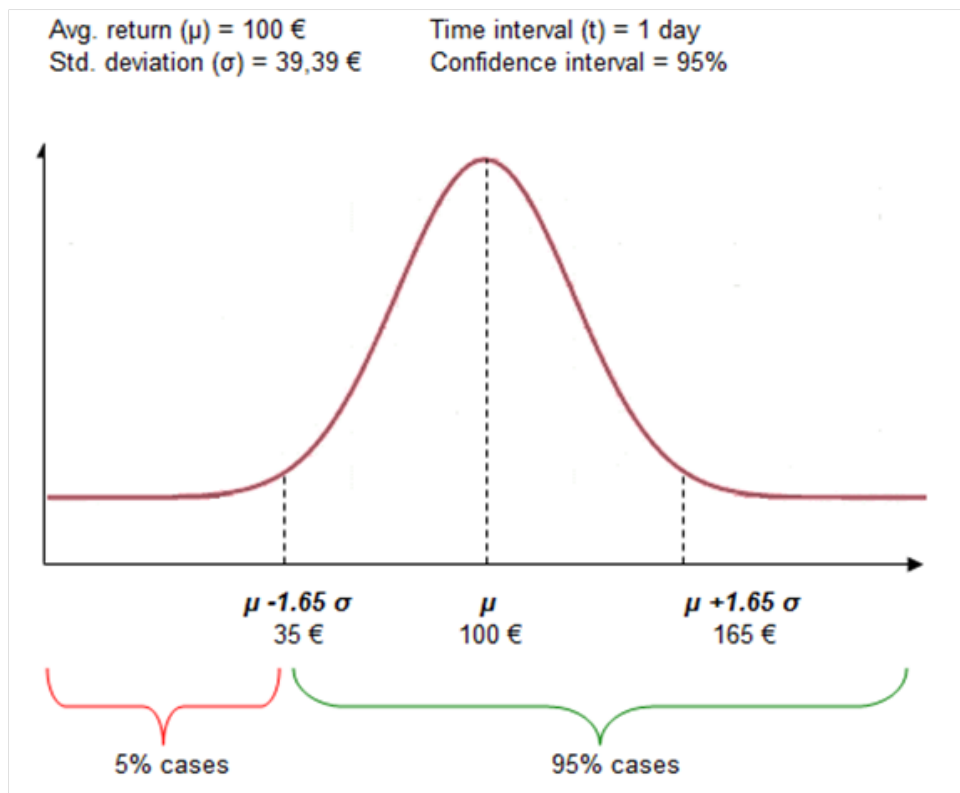
The time horizon chosen to calculate VaR is in accordance with the investment duration or with the minimum time length needed to disinvest in case of loss. The VaR is used also, and perhaps mostly, to determine the minimum capital requirements necessary to compensate for losses resulting from market risk. This measure ap-

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plies, therefore, anytime an evaluation of market risk is done for equity, **bonds**, foreign currencies or derivatives portfolios.

Let's calculate, for example, the VaR during t_0 - t_{0+n} of a portfolio composed by only one financial asset whose returns are distributed as a Normal with mean $\mu = 100$ € and standard deviation $\sigma = 39,39$ €. Assume also that around the interval $\mu \pm 1,65\sigma$ (i.e. from 35 € to 165 €) is distributed 90% of returns. If, at the time of its evaluation, the portfolio is quoted precisely at its equilibrium price (100 € as reported in Fig 1), we would have a VaR (highest loss expected) of -65 € with a 95% confidence level. In other words, between t_0 e t_{0+n} the value of financial asset would be lower than 35 € in 5% of cases and higher in the remaining 95%, as shown in Fig. 1.

Fig. 1: VaR identification and calculation



Given the market risks to which a bank is subject, the vigilance authority requires the maintenance of a minimum level of capital to face those risks. In the case in which a

bank does not have models for VaR calculation (advanced models), developed by internal risk management departments and validated by professionals in charge of supervision (e.g., national central banks), the VaR models applied to derive the requirements can be those suggested by the reference regulation of **Basel III** (standard models).

Major VaR calculation methodologies are a) the Historical-simulation approach, b) the Delta-Normal approach, c) the Monte Carlo approach.

a) Historical simulation approach

Historical simulation approach for VaR calculation bases the future distribution of the asset returns on its past behaviour. Indeed, it estimates the distribution of future returns by starting from a finite number of past observations. This methodology identifies VaR as the x%-quantile of the historical returns distribution for the financial asset. This is the simplest VaR methodology to be used, since it only needs, as an input, the time series data for the asset price observations, and assumes that the future will replicate past behaviour. As a result, no hypothesis on the probability distribution for future returns is necessary. Once built the time series for the asset returns, VaR is identified as the left tail corresponding to the chosen confidence level. This chosen confidence level assures that on x% cases the future asset returns will be higher than calculated VaR, while only on 1-x% cases the future asset returns will be lower than VaR itself. The weakness point of this calculation methodology is represented by the lack of predictive factors into the VaR estimation, being this only based on its past occurrences. In case of use of historical simulation methods for VaR calculation, in order to quantify market risk, Basel Committee requires - to ensure adequate consistency of results - the use of time series having at least a year of daily observations.

b) Normal approach

It is based on the assumption that each asset composing the portfolio has returns following a normal distribution. As a result, the overall portfolio probability distribution

of returns will be a linear combination of the single-asset returns distributions. Consequently, the standard deviation of portfolio returns depends on the standard deviations of its individual components, and on their correlations. The estimate of correlations between individual input factors of the model could result difficult due to the lack of a liquid exchange market for one or more assets (and as a consequence, an unreliable statistical distribution for those factors), or due to the poor quality of historical data used for the estimation. In these cases, one can rely on the use of a single parameter, instead of many, which reasonably approximates all the elements of variability originally considered (for example, instead of considering n equities eligible to affect the return of the financial asset, we will consider only the average historical return of the **stock market** in which all the n shares are exchanged). The simplification of the number of n factors in the model through the use k ($<n$) factors is also known as “risk mapping” activity. The limit of normal approach for VaR calculation is represented by the normality assumption of all the parameters into the model, an hypothesis that is almost never in line with the real situation.

c) Montecarlo simulation approach

It is based on a concept similar to that of the historical simulation approach. It starts from historical data to define what is the most suitable probability distribution to describe past returns behaviour.

- the first step is to identify and estimate input factors affecting financial assets performance; for each of these, an hypothesis about its probability distribution will be formulated;
- subsequently, these parameters will be correlated one with the other through the formulation of a mathematical model where the input parameters are the independent variables while financial asset returns are the dependent variable;
- after the choice of the distribution that best fits the curve of returns, it is used a pseudo-random number generator to create hundreds, or thousands, of possible

evolution scenarios for the underlying factors of the model, resulting in a random distribution of the returns of the asset;

- the VaR will be finally calculated from the generated random distribution.

The main weakness point of this calculation methodology is that choosing the statistical distribution to estimate parameters is not always an easy process. Similarly, the calculation of VaR by using this methodology (since it could depend on many factors) may require long processing times.

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